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Algorithm and Theorem on Student Activity Problems

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Abstract : In this paper, Student Activity Problems are defined. The problems are about joint activities among $n \geq 2$ schools. There are $p > n$ different activity rooms and each school provide p students to join the activities. In each day, each student has to join an activity with a condition that in each room there are n students from n different schools. It is required that each student participate all p activities in p days and each has a chance to work only once with every student from other schools. An algorithm is proposed for such arrangement and a related theorem is also given.

Keywords : Kirkman school girl problems, Steiner triple systems, Student activity problems.
2010 Mathematics Subject Classification : 05B99.

1 Student Activity Problems

There are some studies involving with arrangements or partitions of students (or elements of sets) with some conditions. Studies on the well known Steiner triple systems, see [1] and [2] for examples, and on Kirkman school girl problems, see [1], [2], [3], and [4], provide some questions and answers for some arrangements. There still are varieties of interesting arrangements that have not been investigated.

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In this paper we define Student Activity Problems (SAP). The problems are about arranging joint activities for students from $n \geq 2$ schools. Each school provide $p > n$ students to participate p different activities. The schools provide p rooms for the p activities. In each day each student shall join an activity with a condition that in each room there are n students from n different schools. Each student is required to participate all p activities in p days with the condition that each must have a chance to work only once with every student from other schools.

For example, let S_1, S_2, and S_3 be sets of 4 students from schools s_1, s_2, and s_3 respectively:

$$S_1 = \{1, 2, 3, 4\}$$
$$S_2 = \{5, 6, 7, 8\}$$
$$S_3 = \{9, 10, 11, 12\}$$

Let r_1, r_2, r_3, r_4 be four different activity rooms. For each of the four days Day1, Day2, Day3, and Day4, in each room 3 students from the three different schools shall come to do a joint activity. Let R_1, R_2, R_3, R_4 be sets of students in r_1, r_2, r_3, r_4 respectively. For Day 1, one possible way to arrange students is as follow:

$$R_1 = \{1, 5, 9\}$$
$$R_2 = \{2, 6, 10\}$$
$$R_3 = \{3, 7, 11\}$$
$$R_4 = \{4, 8, 12\}$$

In Figure 1.1, we provide arrangements for Day 1, Day 2, Day 3, and Day 4 such that the conditions of SAP are satisfied. That is, each student participate all 4 activities in 4 days, and each has a chance to work only once with every student from other schools.

	Day1			Day2			Day3			Day4		
R_1	1	5	9	2	7	12	3	8	10	4	6	11
R_2	2	6	10	1	8	11	4	7	9	3	5	12
R_3	3	7	11	4	5	10	1	6	12	2	8	9
R_4	4	8	12	3	6	9	2	5	11	1	7	10

Figure 1.1

In section 2, we provide two cases on SAP. An algorithm for the arrangements of the two cases and for some other general cases are given in section 3. In section 4, we propose a related theorem for the algorithm.

2 Two cases on Student Activity Problems

Consider the following case when $n = 4$ with $p = 5$. Let S_1, S_2, S_3, and S_4 be sets of 5 students from 4 schools s_1, s_2, s_3, and s_4 respectively:

$$
\begin{aligned}
S_1 &= \{1, 2, 3, 4, 5\} \\
S_2 &= \{6, 7, 8, 9, 10\} \\
S_3 &= \{11, 12, 13, 14, 15\} \\
S_4 &= \{16, 17, 18, 19, 20\}
\end{aligned}
\tag{2.1}
$$

Let r_1, r_2, r_3, r_4, and r_5 are five activity rooms. For each of of the five days, in each room four students from four different schools come to do a joint activity. Let R_1, R_2, R_3, R_4, and R_5 be sets of students in rooms r_1, r_2, r_3, r_4, and r_5 respectively. For the first day Day1, one way to arrange students is as follow:

$$
\begin{aligned}
R_1 &= \{1, 6, 11, 16\} \\
R_2 &= \{2, 7, 12, 17\} \\
R_3 &= \{3, 8, 13, 18\} \\
R_4 &= \{4, 9, 14, 19\} \\
R_5 &= \{5, 10, 15, 20\}
\end{aligned}
$$

It takes five days for each student to participate all five activities. One possible arrangement for each room in five days is as follow:

	Day1				Day2				Day3			
R_1	1	6	11	16	5	9	13	17	4	7	15	18
R_2	2	7	12	17	1	10	14	18	5	8	11	19
R_3	3	8	13	18	2	6	15	19	1	9	12	20
R_4	4	9	14	19	3	7	11	20	2	10	13	16
R_5	5	10	15	20	4	8	12	16	3	6	14	17

	Day4				Day5			
R_1	3	10	12	19	2	8	14	20
R_2	4	6	13	20	3	9	15	16
R_3	5	7	14	16	4	10	11	17
R_4	1	8	15	17	5	6	12	18
R_5	2	9	11	18	1	7	13	19

Figure 2.1

Another example is when $n = 6$ with $p = 7$. Let $S_1, S_2, S_3, S_4, S_5, S_6$ be sets of 7 students of six schools $s_1, s_2, s_3, s_4, s_5, s_6$ respectively.

$$S_1 = \{1, 2, 3, 4, 5, 6, 7\}$$
$$S_2 = \{8, 9, 10, 11, 12, 13, 14\}$$
$$S_3 = \{15, 16, 17, 18, 19, 20, 21\}$$
$$S_4 = \{22, 23, 24, 25, 26, 27, 28\}$$
$$S_5 = \{29, 30, 31, 32, 33, 34, 35\}$$
$$S_6 = \{36, 37, 38, 39, 40, 41, 42\}$$

One of possible arrangements is shown in Figure 2.2

	Day1						Day2					
R_1	1	8	15	22	29	36	7	13	19	25	31	37
R_2	2	9	16	23	30	37	1	14	20	26	32	38
R_3	3	10	17	24	31	38	2	8	21	27	33	39
R_4	4	11	18	25	32	39	3	9	15	28	34	40
R_5	5	12	19	26	33	40	4	10	16	22	35	41
R_6	6	13	20	27	34	41	5	11	17	23	29	42
R_7	7	14	21	28	35	42	6	12	18	24	30	36
	Day3						Day4					
R_1	6	11	16	28	33	38	5	9	20	24	35	39
R_2	7	12	17	22	34	39	6	10	21	25	29	40
R_3	1	13	18	23	35	40	7	11	15	26	30	41
R_4	2	14	19	24	29	41	1	12	16	27	31	42
R_5	3	8	20	25	30	42	2	13	17	28	32	36
R_6	4	9	21	26	31	36	3	14	18	22	33	37
R_7	5	10	15	27	32	37	4	8	19	23	34	38

	Day5						Day6					
R_1	4	14	17	27	30	40	3	12	21	23	32	41
R_2	5	8	18	28	31	41	4	13	15	24	33	42
R_3	6	9	19	22	32	42	5	14	16	25	34	36
R_4	7	10	20	23	33	36	6	8	17	26	35	37
R_5	1	11	21	24	34	37	7	9	18	27	29	38
R_6	2	12	15	25	35	38	1	10	19	28	30	39
R_7	3	13	16	26	29	39	2	11	20	22	31	40

	Day7					
R_1	2	10	18	26	34	42
R_2	3	11	19	27	35	36
R_3	4	12	20	28	29	37
R_4	5	13	21	22	30	38
R_5	6	14	15	23	31	39
R_6	7	8	16	24	32	40
R_7	1	9	17	25	33	41

Figure 2.2

One can try the arrangements similar to the tables in Figure 1.1, Figure 2.1, and Figure 2.2 and could find out quickly that without proper algorithm the arrangement could be quite labourious.

In section 3 we provide an algorithm that can be used in arrangements for some general cases.

3 An algorithm for some general cases

Next, we explain an algorithm that is used for the two cases when $n = 4$ with $p = 5$, and $n = 6$ with $p = 7$ in section 2. After that it will become easier in using and proving the algorithm for some other general values of n's and p's.

For the case when $n = 4$ with $p = 5$. For Day1, according to (2.1) we can readily choose the first elements of S_1, S_2, S_3, and S_4 to work in room r_1, i.e.

$$R_1 = \{1, 6, 11, 16\}$$

We can choose the second elements of S_1, S_2, S_3, and S_4 to work in room r_2, i.e.

$$R_2 = \{2, 7, 12, 17\}$$

Similarly, we can obtain

$$R_3 = \{3, 8, 13, 18\}$$
$$R_4 = \{4, 9, 14, 19\}$$
$$R_5 = \{5, 10, 15, 20\}.$$

Writing the above arrangement in the table form, we have Figure 3.1.

	Day1			
R_1	1	6	11	16
R_2	2	7	12	17
R_3	3	8	13	18
R_4	4	9	14	19
R_5	5	10	15	20

Figure 1:

Consider students $1, 2, 3, 4, 5$ in Figure 2.1, i.e. the students in the first column of the arrangement of all five days. We can express the arrangements for the students $1, 2, 3, 4, 5$ in circular forms:

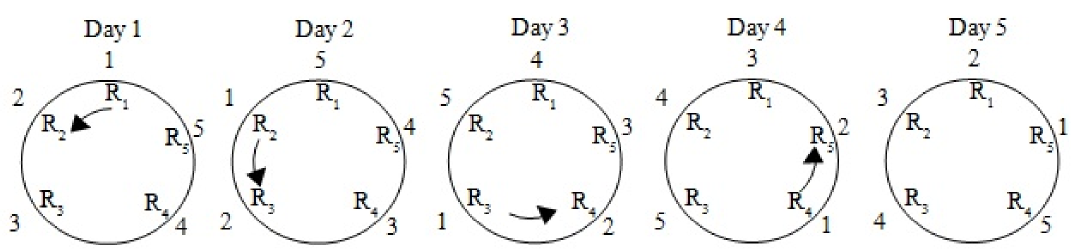

Figure 2:

From Figure 2, we can see that the arrangement on Day 2 can be obtained from Day 1 by shifting each student on Day 1 counterclockwise to the next room (having one step shifting). Also, we can obtain the arrangement on Day 3 from Day 2 by shifting each student on Day 2 counterclockwise to the next room. Similarly, we can obtain the arrangements of Day 4, and Day 5.

Consider the students $6, 7, 8, 9, 10$ in the Figure 2.1, i.e. the students in the second columns of the arrangements of all five days.

We can express the arrangements for the students $6, 7, 8, 9, 10$ in circular forms:

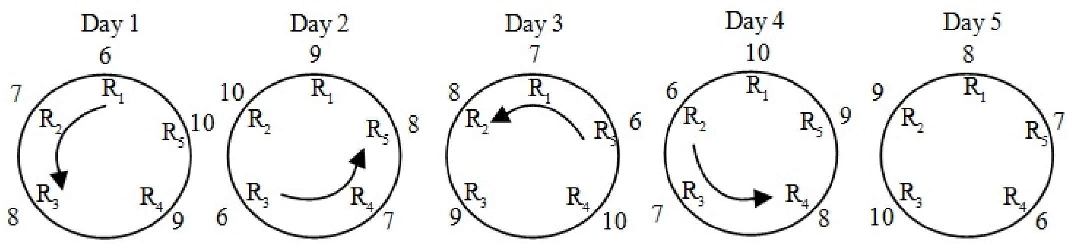

Figure 3:

From Figure 3, we can see that the arrangement on Day 2 can be obtained from Day 1 by shifting each student on Day 1 counterclockwise to the next second room (having two step shifting). By having two step shifting successively we can have the arrangements of students $6, 7, 8, 9, 10$ for Day 3, Day 4, and Day 5.

Consider students $11, 12, 13, 14, 15$ in Figure 2.1, i.e. the students in the third column of the arrangements of all five days.

We can express the arrangements for the students $11, 12, 13, 14, 15$ in circular forms:

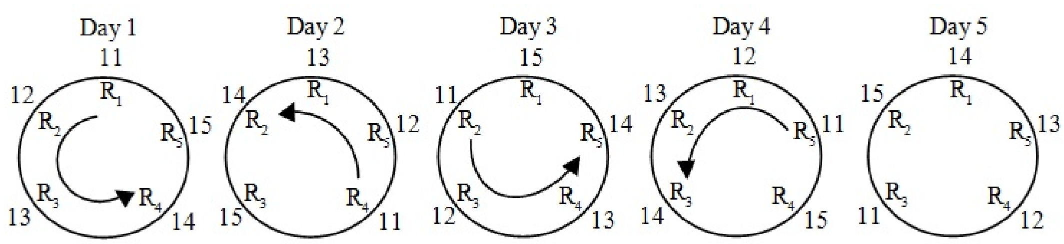

Figure 4:

From Figure 4, we can see that the arrangement on Day 2 can be obtained from Day 1 by shifting each student on Day 1 counterclockwise to the next third room (having three step shifting). By having three step shifting successively we can have the arrangement of students $11, 12, 13, 14, 15$ for Day 3, Day 4, and Day 5.

For the students $16, 17, 18, 19, 20$ in Figure 2.1, i.e. students in the fourth column of the arrangements of all five days. Similarly, starting from Day 1, by shifting each student counterclockwise to the next fourth room (having four step shifting) we can successively obtain the arrangement for Day 2, Day 3, Day 4, and Day 5, see Figure 5

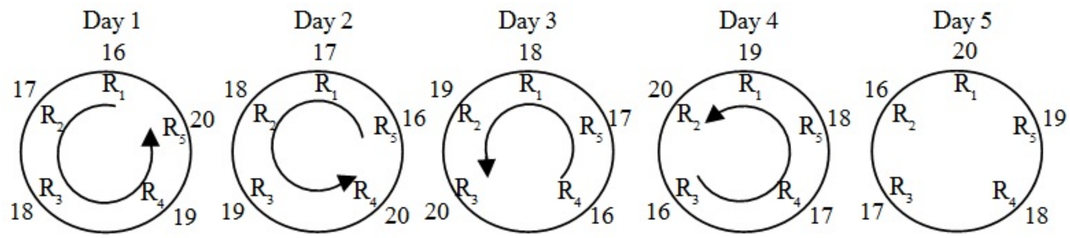

Figure 5:

For the case $n = 6$ with $p = 7$, similar to the case $n = 4$ with $p = 5$, we can readily obtain the arrangement for Day 1 as in Figure 2.2. Also, by expressing the arrangements of Day 1 in circular forms, we can obtain the arrangements of Day 2 from Day 1 by shifting counterclockwise the elements in the first, second, third, $\dots$, sixth columns of Day 1 to the next room, the next second room, the next third room, $\dots$, the next sixth room respectively. Repeating the above process successively and we can obtain the arrangements for Day 3, Day 4, $\dots$, Day 7 respectively.

We can verify directly that the arrangements in Figure 2.1 and Figure 2.2 satisfy all conditions of SAP. In fact, in next section, we show that similar algorithm can work for some other general values of n and p.

4 Sufficient conditions for the use of the algorithm

In section 3, we have shown an algorithm that can be applied for the case when $n = 4$ with $p = 5$, and the case when $n = 6$ with $p = 7$. Similar algorithm can be readily used for the case $n = 2$ with $p = 3$. That is, if

$$S_1 = \{1, 2, 3\}$$
$$S_2 = \{4, 5, 6\}$$

be sets 3 students from 2 schools with $p = 3$. We can obtain the arrangement as in Figure 6

	Day 1		Day 2		Day 3	
R_1	1	4	3	5	2	6
R_2	2	5	1	6	3	4
R_3	3	6	2	4	1	5

Figure 6:

We have seen that the arrangement in Figure 2.1, Figure 2.2, and Figure 6 are based on the same algorithm, but we note that for the case $n = 3$ with $p = 4$ the arrangements in Figure 1.1 are not from the algorithm.

Next, we propose the algorithm in general form, and after that we provide a related theorem for the algorithm. For refering, this algorithm shall be called School Activity Algorithm(SAA).

Let $s_1, s_2, \ldots, s_n$ be n schools and $a_{i1}, a_{i2}, \ldots, a_{ip}$ be p students from school s_i. Let $S_1, S_2, \ldots, S_n$ be the sets of students from schools $s_1, s_2, \ldots, s_n$ respectively. That is

$$\begin{aligned} S_1 &= \{a_{11}, a_{12}, a_{13}, \ldots, a_{1p}\} \\ S_2 &= \{a_{21}, a_{22}, a_{23}, \ldots, a_{2p}\} \\ S_3 &= \{a_{31}, a_{32}, a_{33}, \ldots, a_{3p}\} \\ &\vdots \\ S_n &= \{a_{n1}, a_{n2}, a_{n3}, \ldots, a_{np}\} \end{aligned}$$

According to a condition of SAP, one possible arrangement for Day 1 can be as follow:

	Day 1				
R_1	a_{11}	a_{21}	a_{31}	$\ldots$	a_{n1}
R_2	a_{12}	a_{22}	a_{32}	$\ldots$	a_{n2}
R_3	a_{13}	a_{23}	a_{33}	$\ldots$	a_{n3}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
R_p	a_{1p}	a_{2p}	a_{3p}	$\ldots$	a_{np}

Figure 7:

As in the examples in section 3, we can express the arrangements on Day 1 of Figure 7 in circular forms:

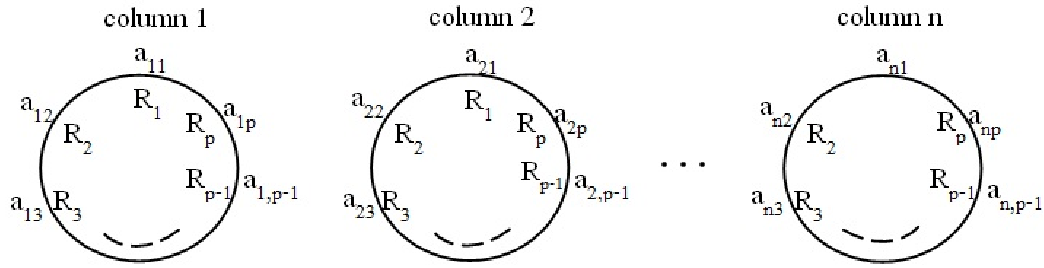

Figure 8:

We can obtain the arrangement on Day 2 from Day 1 by shifting counterclockwise the students in the first, second, third, ..., $(n-1)th$ column of Day 1 in Figure 7 to the next room, the next second room, the next third room, ..., the next $(n-1)th$ room respectively, see Figure 4.4.

	Day 2			
R_1	a_{1p}	$a_{2,p-1}$	$\dots$	a_{n2}
R_2	a_{11}	a_{2p}	$\dots$	a_{n3}
R_3	a_{12}	a_{21}	$\dots$	a_{n4}
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
R_p	$a_{1,p-1}$	$a_{2,p-2}$	$\dots$	a_{n1}

Figure 4.4

Repeating the process successively, we can obtain the arrangements for Day 3, Day 4, ..., and Day p respectively, see Figure 4.5.

	Day 3			
R_1	$a_{1,p-1}$	$a_{2,p-3}$	$\dots$	a_{n3}
R_2	a_{1p}	$a_{2,p-2}$	$\dots$	a_{n4}
R_3	a_{11}	$a_{2,p-1}$	$\dots$	a_{n5}
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
R_p	$a_{1,p-2}$	$a_{2,p-4}$	$\dots$	a_{n2}

$\dashrightarrow$

	Day p			
R_1	a_{12}	a_{23}	$\dots$	a_{np}
R_2	a_{13}	a_{24}	$\dots$	a_{n1}
R_3	a_{14}	a_{25}	$\dots$	a_{n2}
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
R_p	a_{11}	a_{22}	$\dots$	$a_{n,p-1}$

Figure 4.5

Next, we propose Theorem 4.1 that provide sufficient conditions by which the algorithm can be used.

Theorem 4.1. *Let $n \geq 2$ be number of schools each of which provide $p > n$ students to participate p joint activities. For any prime number $p > n$, the algorithm SAA provide arrangements that satisfy all conditions of SAP.*

Proof. Let $S_1, S_2, \dots, S_n$ be sets of p students from n schools as defined earlier in this section. Consider the arrangement of students a_{ij}'s in Day 1 in Figure 7 and Figure 8. Next, consider Figure 4.6 that show the numbers of step shiftings of each column of students in shifting from one room to another room in accordance with the algorithm SAA.

	Numbers of step shiftings on Day i's after shifting from Day 1				
	a_{1i}	a_{2i}	a_{3i}		a_{ni}
Day 1	0	0	0	$\dots$	0
Day 2	1	2	3	$\dots$	n
Day 3	2	4	6	$\dots$	2n
Day 4	3	6	9	$\dots$	3n
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
Day p	$p-1$	$2(p-1)$	$3(p-1)$	$\dots$	$n(p-1)$

Figure 4.6

To simplify the proof, Figure 4.6 is rewritten and shown in Figure 4.7.

	Numbers of step shiftings on Day i's after shifting from Day 1				
	a_{1i}	a_{2i}	a_{3i}		a_{ni}
Day 1	0	0	0	$\dots$	0
Day 2	1.1	1.2	1.3	$\dots$	1.n
Day 3	2.1	2.2	2.3	$\dots$	2.n
Day 4	3.1	3.2	3.3	$\dots$	3.n
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
Day p	$(p-1).1$	$(p-1).2$	$(p-1).3$	$\dots$	$(p-1)n$

Figure 4.7

According to the conditions of SAP, it is required that no pairs of students shall appear together twice in the arrangement. We claim that the algorithm used, with the condition that p is prime number, do not allow any pair of students to appear together twice. Since any pair of students in different columns (different schools) shift with different step shiftings in circular manner, should any pair of students appear together for the second time in the arrangement, their differences in step shiftings must be equal to some multiple of p. This could not happen because, from Figure 4.7 with $n < p$, we can see that all differences of step shiftings of Day 2, Day 3, Day 4, $\dots$, Day p are not multiple of p. Therefore, any pair of students in all $R_1, R_2, \dots, R_p$ of Day 1 shall has no chance to work together again in all of the following days. Next, consider the arrangement in $R_1, R_2, \dots, R_p$ of Day 2. With similar reasons we can conclude that no pair of students in $R_1, R_2, \dots, R_p$ of Day 2 shall has no chance to work together again in all of the following days. Next, consider the arrangement in $R_1, R_2, \dots, R_p$ of Day 3. Again with similar reasons we can conclude that no pairs of students in $R_1, R_2, \dots, R_p$ of Day 3 shall have no chance to work together in the following days. Follow the same reasons, finally, we conclude that no pair of students work together more than one time. Also, since each day a student work with $(n-1)$ students from other $(n-1)$ schools, therefore after p days each student shall has a chance to work with $p(n-1)$ students, i.e. all students from other schools. Therefore the conditions of SAP are satisfied and so the theoem is proved. □

From the theorem, we note that the condition that p is prime is a sufficient condition, that is the arrangement is possible when treated with the algorithm.

For examples, when $n = 3, 5, 7$ which are prime numbers, the theorem predict that the arrangements for these cases are possible when the algorithm is used. We have seen some cases of the arrangements in Figure 6, Figure 2.1, and Figure 2.2.

Also from the theorem, we note that the algorithm can be used for any integer n such that $2 \leq n < p$.

For example, when $p = 7$, the arrangement in Figure 2.2 can also work for all $n = 2, 3, 4, 5, 6$. Suppose $p = 7$ with $n = 4$, from Figure 2.2 we can have the required arrangement by consider, on each Day i arrangement, only students in column 1, column 2, column 3, and column 4.

If p is not prime the arrangements that satisfy the conditions of SAP may still be possible for some cases. For example, the arrangements in Figure 1.1 are for the case when $n = 3$ with $p = 4$, but we can see that the arrangements are not from the algorithm SAA we use for the cases when p are prime numbers. The studies for the cases when p are not prime are open for further investigations.

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