Trading Gold Future with ARIMA-GARCH model

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Abstract: In this paper, we forecast volatility of gold prices using ARIMA-GARCH models. All models are estimated under three distributional assumptions which are Normal, Student-t and GED. The gold price log returns are stationary. We found that the ARIMA(2,0,2) gave the best performance model for forecasting the return of gold. Serial correlation in the squared returns suggests conditional heteroskedasticity. This empirical part adopts GARCH models to estimate the volatility of the gold price. To account for fat-tailed features of financial returns, we consider three different distributions for the innovations. The trading details we have used describe forecasts of a closed price of gold price and trading in the gold future contract (GF10J16). We found that the cumulative of return with ARIMA(2,0,2)-GARCH-N model and the ARIMA(2,0,2)-GARCH-GED model give cumulative of return more than the ARIMA(2,0,2)-GARCH-t models.

Keywords: forecasting, volatility, gold price, ARIMA-GARCH.

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1 Introduction

The characteristic that all financial markets have in common is uncertainty, which is related to their short and long-term price state. This feature is undesirable for the investor but it is also unavoidable whenever the financial market is selected as the investment tool. The best that one can do is to try to reduce this...
uncertainty. Financial market forecasting (or Prediction) is one of the instruments in this process.

The financial market forecasting task divides researchers and academics into two groups: those who believe that we can devise mechanisms to predict the market and those who believe that the market is efficient and whenever new information comes up the market absorbs it by correcting itself, thus there is no space for prediction. Furthermore, they believe that the financial market follows a random walk, which implies that the best prediction you can have about tomorrow’s value is today’s value.

In time series, a financial price transformed to log return series for the stationary process which looks like white noise. Mehmet [1] said financial returns have three characteristics. First is volatility clustering that means large changes tend to be followed by large changes and small changes tend to be followed by small changes. Second is fat-tailedness (excess kurtosis) which means that financial returns often display a fatter tail than a standard normal distribution and the third is leverage effect which means that negative returns result in higher volatility than positive returns of the same size.

The generalised autoregressive conditional heteroskedasticity (GARCH) models mainly capture three characteristics of financial returns. The development of GARCH type models was started by Engle [2]. Engle introduced ARCH to model the heteroskedasticity by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev [3] generalised the ARCH (GARCH) model by modelling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance.

Among these models, the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle [2] and its extension; Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model by Bollerslev [3], and Taylor [4] were found to be the first models introduced into the literature and have become very popular in that they enable the analysts to estimate the variance of a series at a particular point in time Enders [5]. Since then, there have been a great number of empirical applications of modelling the conditional variance of a financial time series (See for example, Nelson [6], Bollerslev et al. [7], Engle and Patton [8], Shin [9], Alberg et al. [10], Shamiri and Isa [11] and Kalu [12]. These types of models were designed to explicitly model and forecast the time-varying conditional second order moment (variance) of a series by using past unpredictable changes in the returns of that series, and have been applied successfully in economics and finance, but more predominantly in financial market research.

Gold is a precious metal which is also classed as a commodity and a monetary asset. Gold has acted as a multifaceted metal through the centuries, possessing similar characteristics to money in that it acts as a store of wealth, a medium of exchange and a unit of value. Gold has also played an important role as a precious metal with significant portfolio diversification properties. Gold is used in industrial components, jewellery and as an investment asset. The quantity of gold required is determined by the quantity demanded industry investment and jewellery use. Therefore an increase in the quantity demanded by the industry
will lead to an increase in the price of the metal.

The changing price of gold can also be the result of a change in the Central Banks holding of these precious metals. In addition, changes in the rate of inflation, currency markets, political harmony, equity markets, and producer and supplier hedging, all affect the price equilibrium.

Gold Futures is an alternative investment tool which relies on the gold price movement. The investors can benefit from the gold futures investment by making the profit from both directions, either up or down, which is like Stock Index Futures trading. In addition, Gold Futures can also hedge against gold price fluctuations or stock market volatility due to the negative correlation to the stock market. This will provide a greater opportunity to make a profit when the stock market declines during an economic downturn.

Gold Futures in Thailand are futures contracts which rely on gold bullion with a purity of 96.5 percent due to its popularity among buyers nationwide for gold physical trading. Gold Futures trade in implement cash settlement method with no need of physical delivery.

In this paper, we use ARIMA-GARCH models to forecast the volatility of gold prices. Moreover, we shall use this estimated volatility to forecast the closing price of gold. Finally, we apply the forecasting price to the gold price for trading in gold future contracts with a maturity date of April 2016 (GF10J16).

In the next section, we present the ARIMA-GARCH model. Estimation and in-sample evaluation results are given in section 3. In section 4, we apply the forecasting price to the gold price for trading in future contracts. The conclusion is given in section 5.

2 ARIMA-GARCH Model.

Let $P_t$ denote the series of the financial price at time $t$ and $\{r_t\}_{t>0}$ be a sequence of random variables on a probability space $(\Omega, F, P)$. For index $t$ denotes the daily closing observations and $t = -R+1, \ldots, n$. The sample period consists of an estimation (or in-sample) period with $R$ observations ($t = -R+1, \ldots, 0$), and an evolution (or out-of-sample) period with $n$ observations ($t = 1, \ldots, n$), let $r_t$ be the logarithmic return (in percent) on the financial price at time $t$ i.e.

$$r_t = 100 \cdot \ln(\frac{P_t}{P_{t-1}})$$

2.1 ARMA Models

We can have combinations of the two processes to give a new series of models called $ARMA(p, q)$ models. The general form of the ARMA($p$, $q$) models is following (Tsay, [13]):

$r_t = \varphi_1 r_{t-1} + \varphi_2 r_{t-2} + \ldots + \varphi_p r_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}$
which can be rewritten, using the summations as:

\[ r_t = \sum_{i=1}^{p} \varphi_i r_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \]

or, using the lag operator:

\[ r_t (1 - \varphi_1 L - \varphi_2 L^2 - \ldots - \varphi_p L^p) = (1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_p L^q) \varepsilon_t \]

\[ \Phi(L) r_t = \Theta(L) \varepsilon_t \]

In the ARMA(p, q) model the condition for stationarity has to deal with the AR(p) part of the specification only.

\section*{2.2 Integrated Processes and the ARIMA Models: ARIMA Models}

An integrated series the ARMA(p, q) model can only be made on time series \( r_t \) that stationary. In order to avoid this problem, and in order to induce stationarity, we need to detrend the raw data through a process called \textit{differencing}

\[ \Delta r_t = r_t - r_{t-1}. \]

As most economic and financial time series show trends to some degree, we nearly always end up taking first differences of the input series. If, after first differencing, a series is stationary then the series is also called \textit{integrated to order one}, and denoted \( I(1) \).

If a process \( r_t \) has an ARIMA(p, d, q) representation, the has an ARMA(p,q) representation as presented by the equation below:

\[ \Delta^d r_t (1 - \varphi_1 L - \varphi_2 L^2 - \ldots - \varphi_p L^p) = (1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_p L^q) \varepsilon_t \]

In general Box-Jenkins popularised a three-stage method aimed at selecting an appropriate (parsimonious) ARIMA model for the purpose of estimating and forecasting a univariate time series. Three stages are:

- **Identification:** A comparison of the sample ACF and PACF to those of various theoretical ARIMA processes may suggest several plausible models. If the series is non-stationary the ACF of the series will not die down or show signs of decay at all. A common stationarity-inducing transformation is to take logarithms and then first differences of the series. Once we have achieved stationarity, the next step is to identify the p and q orders of the ARIMA model.

- **Estimation:** In this second stage, the estimated model are compared using AIC and SBC.

- **Diagnostic checking:** In the diagnostic checking stage we examine the goodness of fit of the model. We must be careful here to avoid overfitting (the procedure of adding another coefficient in appropriate). The special statistics that we use here is the Box-Piece statistic (BP) and the Ljung-Box (LB) Q-statistic, which serve to test for autocorrelations of the residual.
3 GARCH Model

The generalised autoregressive conditional heteroskedasticity (GARCH) model, developed by Engle [2] and Bolleslev [3], has been proven to be a useful tool to empirically capture the momentum in conditional variance. Under GARCH, shocks to variance persist according to an autoregressive moving average (ARMA) structure of the squared residuals of the process. The GARCH(p,q) model allows the conditional variance of the random disturbance to depend linearly on the past behaviour of the squared errors. Moreover, the GARCH(1,1) specification has proven to be an adequate representation for most financial time series. The GARCH (1,1) model for the series of the returns \( r_t \) can be written as (Marcucci, [14])

\[
\begin{align*}
  r_t &= \mu_t + \varepsilon_t = \mu_t + \eta_t \sqrt{h_t}; \\
  h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}; \\
  \eta_t &= \text{i.i.d. process with zero mean and unit variance.}
\end{align*}
\]

where \( \mu_t \) is ARIMA process of \( r_t \), \( \alpha_0 > 0, \alpha_1 \geq 0 \) and \( \beta_1 \geq 0 \) are assumed to be non-negative real constants to ensure that \( h_t \geq 0 \). We assume \( \eta_t \) is an i.i.d. process with zero mean and unit variance.

4 Forecasting the Gold Price and Volatility

We forecast financial price at k-step-ahead with ARIMA-GARCH models. Denote \( \hat{r}_{t,t+k} \) as k-step-ahead forecasting logarithm return of financial price at time \( t \) depend on \( F_{t-1} \). We compute as: The general form of the ARMA(p, q) models is following:

\[
\begin{align*}
  r_t &= \varphi_1 r_{t-1} + \varphi_2 r_{t-2} + \ldots + \varphi_p r_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \\
  \hat{r}_{t,t+k} &= E_{t-1} [r_{t+k}] = \varphi_1 \hat{r}_{t+k-1} + \varphi_2 \hat{r}_{t+k-2} + \ldots + \varphi_p \hat{r}_{t+k-p} \\
  \text{Forecasting financial price one-step-ahead, we combine in log-return of financial price is} \\
  \hat{P}_{t+1} &= P_t \cdot \exp \left[ \frac{\varphi_1 \hat{r}_{t+k-1} + \varphi_2 \hat{r}_{t+k-2} + \ldots + \varphi_p \hat{r}_{t+k-p}}{100} \right].
\end{align*}
\]

Forecasting financial price one-step-ahead, we use combine in log-return of financial volatility of GARCH(1,1) is \( \hat{h}_{t+1} = E(h_t) = \alpha_0 + \alpha_1 E(\varepsilon_{t-1}^2) + \beta_1 E(h_{t-1}) \).
5 Empirical Methodology and Model Estimation Results

5.1 Data

The data set used in this study is the daily closed prices of Gold price ($P_t$) over the period 2/01/2015 through 31/03/2016 ($t = 1, 313$ observations). The data set is obtained from the basis of the London Gold Market Fixing Limited on a day and the foreign exchange rate for Baht to US dollars announced by TFEX (The Thailand Futures Exchange) on a day, after conversion for weight and fineness. The data set is divided into in-sample ($R = 291$ observations) and out-of-sample ($n = 23$ observations). The plot of $P_t$ and log returns series($r_t$) are given in Figure 1. Plot $P_t$ and $r_t$ displays usual properties of financial data series. As expected, volatility is not constant over time and exhibits volatility clustering with large changes in the indices often followed by large changes, and small changes often followed by small changes.

Figure 1: Graph of Gold price closed prices and returns series for the period 2/01/2015 through 31/03/2016.

Descriptive statistics of $r_t$ are represented in Table 2. As Table 2 shows, $r_t$ has a positive average return of 0.0143. The daily standard deviation is 0.99257. The series also displays a negative skewness of 0.328 and an excess kurtosis of 1.507. These values indicate that the returns are not normally distributed, namely it has fatter tails because skewness does not equal zero and kurtosis is less than 3. Also, the Jarque-Bera test statistic of 30.48 confirms the non-normality of $r_t$. And the Augmented Dickey-Fuller test of -6.0134 indicates that $r_t$ is stationary.
5.2 ARIMA Model

Since the data is for the return of gold, Figure 1, shows that the series is stationary. The ACF and PACF of the original data, as shown in Figure 2, show that the return data is stationary. Therefore, an ARIMA \((p, 0, q)\) model could be identified.

After the ARIMA model was identified above, the \(p\) and \(q\) parameters need to be identified for our model. In Figure 2, we have one autoregressive \((p)\) and moving average \((q)\) parameter and the ACF has exponential decay starting at lag 3 and 5. Since we have identified for the return data our tentative model will be ARIMA \((1, 0, 1)\) and ARIMA \((2, 0, 2)\).

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-2.70</td>
</tr>
<tr>
<td>Max</td>
<td>4.36</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0143</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.99257</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.328</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.507</td>
</tr>
<tr>
<td>Jarque-Bera Normality test</td>
<td>30.48 (P-value= 0.000)</td>
</tr>
<tr>
<td>Augmented Dickey-Fuller test</td>
<td>-6.0134 (P-value= 0.01)</td>
</tr>
</tbody>
</table>

Figure 2: ACF and PACF of Gold price closed prices and returns series.
After we fitted ARIMA (1, 0, 1) and ARIMA (2, 0, 2) for our data. We need to estimate the parameters values for our model. As a rule of thumb, in ARIMA modelling we need to minimize the sum squared of the residuals which need to be minimized between the forecasted and existing values. We found the sum squared of the residuals for the model ARIMA (2, 0, 2) was 277.1189 and the values of the parameters are shown in Table 3 as follows:

The ARIMA(2,0,2) gives the best performance for forecasting return of gold. The model as:

$$(1 + 0.27521262B - .80497967B^2)r_t = (1 - 0.37828030B + 0.84936001B^2)\varepsilon_t$$

Figure 3 shows the time series plot of actual return according to ARIMAs forecast of return.

![Figure 3: Graph of Gold price returns series and forecast with ARIMA(2,0,2).](image)
The autocorrelation functions (ACF) test the significance level of autocorrelation in Table 4, when we apply Ljung Box and Q-test. The null hypothesis of the test is that there is no serial correlation in the return series up to the specified lag. Serial correlation in the $P_t$ is confirmed as non-stationary but $r_t$ is stationary. Because the serial correlation in the squared returns is non-stationary this suggests conditional heteroskedasticity. Therefore, we analyse the significance of autocorrelation in the squared mean adjusted return $(r_t - \delta)^2$ series by Ljung-Box Q-test. And apply Engles ARCH test.

<table>
<thead>
<tr>
<th>Lag</th>
<th>ACF of Gold price closed price</th>
<th>ACF of Gold price log return</th>
<th>ACF of Gold price square Results for Engle’s ARCH</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LBQ Test P-value</td>
<td></td>
<td>LBQ Test P-value</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.00  2546.49  0.00</td>
<td>-0.01  0.20  0.66  0.12</td>
<td>38.20  0.00</td>
<td>67.675  0.000</td>
</tr>
<tr>
<td>2</td>
<td>1.00  5086.78  0.00</td>
<td>-0.01  0.57  0.75  0.09</td>
<td>56.49  0.00</td>
<td>67.73  0.000</td>
</tr>
<tr>
<td>3</td>
<td>1.00  7629.94  0.00</td>
<td>0.03  2.63  0.45  0.11</td>
<td>89.66  0.00</td>
<td>69.796  0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.99  10184.81  0.00</td>
<td>-0.12  3.53  0.47  0.06</td>
<td>98.04  0.00</td>
<td>70.644  0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.99  12670.71  0.00</td>
<td>0.01  3.60  0.61  0.08</td>
<td>114.29  0.00</td>
<td>70.695  0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.99  15186.73  0.00</td>
<td>-0.03  5.24  0.51  0.08</td>
<td>130.82  0.00</td>
<td>73.794  0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.99  17686.96  0.00</td>
<td>-0.04  8.83  0.27  0.07</td>
<td>144.15  0.00</td>
<td>76.115  0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.99  20201.63  0.00</td>
<td>-0.02  10.31  0.24  0.07</td>
<td>156.59  0.00</td>
<td>78.019  0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.99  22700.83  0.00</td>
<td>0.02  11.71  0.23  0.14</td>
<td>208.84  0.00</td>
<td>78.603  0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.99  25194.08  0.00</td>
<td>-0.01  12.09  0.28  0.09</td>
<td>227.81  0.00</td>
<td>79.005  0.000</td>
</tr>
</tbody>
</table>

5.3 GARCH Models

This empirical part adopts GARCH models to estimate the volatility of the $P_t$. In order to account for the fat tails feature of financial returns, we consider three different distributions for the innovations: Normal (N), Student-t (t) and Generalised Error Distributions (GED).

Table 5 presents an estimation of the results for GARCH models. It is clear from the table that almost all parameter estimates in GARCH type are highly significant at 5 percent.

Table 5 Summary results of GARCH models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>N 0.3501 t 0.4144 GED 0.3641</td>
</tr>
<tr>
<td>standard error</td>
<td>N 0.3219 t 0.4553 GED 0.4834</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>N 0.03 t 0.0936 GED 0.0181</td>
</tr>
<tr>
<td>standard error</td>
<td>N 0.0449 t 0.0434 GED 0.053</td>
</tr>
<tr>
<td>$\nu$</td>
<td>N 5.8269 t 1.2961 GED 0.4009</td>
</tr>
<tr>
<td>standard error</td>
<td>N 2.2142 t 0.1459 GED 0.1059</td>
</tr>
</tbody>
</table>
6 Trading Future Contract with Forecast Volatility and Forecast Price

The aim of this study is to evaluate the profitability of applying different models to the volatility of gold prices. We assumed the market is a perfect market and the positions in our strategy are not longer than one day as described below. We applied the Bollinger band indicator and we used samples of 23 days from 1 to 31 March 2016 (We trade one contract in GF10J16 series is future contract in gold price with maturity date at April 2016) to trade in one contract with day by day and we did not include settlement, return do not include cost price i.e. margin, fee charged. The strategy, firstly we forecast close price in the trading day \((\hat{C}_{t+1})\) if forecast close price greater than open price \((O_{t+1})\), we will long position. The net daily rate of return for long position is computed as follows (N. Sopipan et al., [15]):

\[
R_{t+1} = C_{t+1} - (O_{t+1} - m \cdot \sqrt{\hat{h}_{t+1}})
\]

where \(R_{t+1}, C_{t+1}, O_{t+1}\) are the return, forecast close, open price, \(\hat{h}_{t+1}\) is forecasting volatility at next day \((t+1)\) and \(m \in \mathbb{Z}^+\) is constants. If forecast close price \((\hat{C}_{t+1})\) less than open price \((O_{t+1})\), we will short position The net daily rate of return on a short position is computed as follows:

\[
R_{t+1} = (O_{t+1} + m \cdot \sqrt{\hat{h}_{t+1}}) - C_{t+1}
\]

Table 6 shows that the cumulative of return with ARIMA(2,0,2)-GARCH-N model and the ARIMA(2,0,2)-GARCH-GED model give cumulative of return more than the ARIMA(2,0,2)-GARCH-t models when we use \(m=35\).

7 Conclusion

In this paper, we forecast volatility of gold prices using ARIMA-GARCH models. All models are estimated under three distributional assumptions which are Normal, Student-t and GED.

We first analyse in-sample performance of various volatility models to determine the best form of the volatility model over the period 2/01/2015 through 31/03/2016. As expected, volatility is not constant over time.

Descriptive statistics of return series are represented by returns with fatter tails. The Augmented Dickey-Fuller test indicates gold price log returns are stationary. Serial correlation in the gold price confirms it is non-stationary but serial log returns of gold price are stationary.

We found that the ARIMA(2,0,2) gives the best performance for forecasting return of gold. Serial correlation in the squared returns suggests conditional heteroskedasticity. This empirical part adopts GARCH models to estimate the volatility of the gold price. In order to account for fat-tailed features of financial
returns, we consider three different distributions for the innovations. Almost all parameter estimates in GARCH models are highly significant at 5 percent.

The trading details we have used describe forecasts of the closed price of the gold price between 1 to 31 March 2016 and trading in gold future contract (GF10J16). We found that the cumulative of return with ARIMA(2,0,2)-GARCH-N model and the ARIMA(2,0,2)-GARCH-GED model give cumulative of return more than the ARIMA(2,0,2)-GARCH-t models when we use m=35.

For further study, three or four volatility regimes setting can be considered rather than two-volatility regimes. Also, using Markov Regime Switching with other volatility models e.g. EGARCH, GJR. In addition, the performance of MRS-GARCH models can be hedged in future for long and short positions.

References


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