Thai Journal of Mathematics
Volume 8 (2010) Number 2 : 373–381

www.math.science.cmu.ac.th/thaijournal
Online ISSN 1686-0209

A New Sequence Space and Matrix Transformations

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Abstract : The main purpose of this paper is to define the sequence space bs(p,s)and determine the necessary and sufficient conditions on the matrix sequence $\mathcal{A} = (A_i)$ in order that $\mathcal{A} \in (X, Y)$, where $X = l_\infty(p, s), bs(p, s)$ *and* $Y = l_\infty, f, bs, fs$.These results are more general than those of Lascarides and Maddox[6],Basar Solak[3],Nanda[10] and Solak[12].

Keywords : Infinite matrices, Almost convergence, $\mathcal{A}$-summability.
2000 Mathematics Subject Classification : 40C05, 40H05, 46A45.

1 Introduction

Let $A = (a_{nk})$ be an infinite matrix of real numbers $a_{nk}(n, k = 0, 1, \cdots)$ and X,Y be two non-empty subsets of the space W of real sequences.We say that A defines a (matrix) transformation from X into Y,and we denote it by A:X→Y, if for every sequence x=(x_k) ∈X,the sequence Ax=$(A_n$(x)) is in Y,where the series $A_n(x) = \sum_k a_{nk}x_k$ converges for all n. By (X,Y) we denote the class of all such matrices.

By l_∞ and bs we denote the spaces of all bounded sequences and bounded series respectively.The shift operator D is defined on l_∞ by $(DX)_n = x_{n+1}$.A Banach limit L is defined on l_∞ as a non-negative linear functional,such that L(Dx)=L(x) and L(e)=1,where e=(1,1,1—)(see[1]).A sequence $x \in l_\infty$ is said to be almost convergent to the generalized limit ℓ if all Banach limits of x is ℓ and denoted by f-limx=ℓ.Lorentz[7] proved that f-limx=ℓ if and only if $lim_m \frac{(x_n+x_{n+1}+\cdots+x_{n-m+1})}{m} = l$ uniformly in n.It is well known that a convergent sequence is almost convergent such that its limit and generalized limit are equal.

Given any infinite series $\sum a_n$,it is said to be almost convergent if its sequence of partial sums is almost convergent.By f and fs we denote the spaces of all almost convergent real sequences and series,respectively.

For a sequence $p = (p_k)$ with $p_k > 0$,the following sequence spaces where defined by Maddox[6,9] and Solak[12],respectively:

$l(p) := \{x : \sum_k |x_k|^{p_k} < \infty\}, \quad 1 < p_k \leq supp_k < \infty,$

$l_\infty(p) := \{x : \sup_k |x_k|^{p_k} < \infty\},$

$bs(p) := \{x : \sup_k |\sum_{n=0}^{k} x_n|^{p_k} < \infty\},$

In [4] and [2],the spaces $l(p)$ and l_∞(p) were extended respectively to $l(p, s)$ and l_∞(p,s) for $s \geq 0$.i.e,

$l(p, s) := \{x : \sum_k k^{-s}|x_k|^{p_k} < \infty\},$

$l_\infty(p, s) := \{x : \sup_k k^{-s}|x_k|^{p_k} < \infty\},$

Here we give an obvious extension of bs(p) to bs(p,s) for $s \geq 0$.i.e.

$bs(p, s) := \{x : \sup_k k^{-s}|\sum_{n=0}^{k} x_n|^{p_k} < \infty\},$

Let $\mathcal{A}$ denote the sequence of real matrices $A_i = (a_{nk}^{(i)})$.We write for a sequence $x = (x_k)$.

$(Ax)_n^i = \sum_k a_{nk}^{(i)} x_k$

if it exists for each n,i and

$\mathcal{A}x = ((Ax)_n^i)_{n,i=0}^\infty$

A sequence x is said to be $\mathcal{A}$-summable to ℓ if $lim_n (Ax)_n^i = l$, uniformly in i.To denote the matrix sequence $\mathcal{A}$ of the class (X,Y), we write $\mathcal{A} \in (X, Y)$.If $a_{nk}^{(i)} = a_{nk}$ for all i, then $\mathcal{A}$ reduces to the usual summability method A and if $a_{nk}^{(i)} = 1(n = k)$ for all i,$= 0(n \neq k)$ for all i.then $\mathcal{A}$ corresponds to the identity matrix I which is equivalent to ordinary convergence.So the method $\mathcal{A}$ is more general than the usual summability method A.

Throughout the sections 2 and 3 by $s = (s_k)$ we denote the sequence of partial sums of the series $\sum x_n$.Thus it is clear that $s \in l_\infty(or f)$,whenever $x \in bs(or fs)$.

2 Matrix sequences from $l_\infty(p.s)$ into $l_\infty.f.bs$ and fs.

Theorem 2.1. *$\mathcal{A} \in (l_\infty(p.s).l_\infty)$ if and only if (2.1) holds.*

$$D(N) = \sup_{n,i} \sum_k k^{\frac{s}{p_k}} |a_{nk}(i)| N^{\frac{1}{p_k}} < \infty \quad \textit{for every integer} \;\; N > 1. \tag{2.1}$$

To prove Theorem 2.1 we require he following result.

Lemma 2.2. *(see Stieglitz[13], Folgerung[1])*
Given $\mathcal{B} = (B_i)$, then he following three statements are equivalent.
(a) $\mathcal{B}x$ exists for all $x \in l_\infty$.
(b) $\mathcal{B}x$ exists for all $x \in c_0$.
(c) $\sum_k |b_{nk}^{(i)}| < \infty, (n.i)$, where c_0 denoes the space of sequences convergent to zero.

Proof of the Theorem 2.1. Necessity. Suppose that $\mathcal{A} \in (l_\infty(p.s).l_\infty)$. If the condition (2.1) is not true, then there exists an integer $N > 1$ such that $D(N) = \infty$. Then, by Lemma 1, the matrix sequence $\mathcal{B}x = (b_{nk}^{(i)}) = (a_{nk}^{(i)} k^{\frac{s}{pk}} N^{\frac{1}{pk}}) \notin (l_\infty, l_\infty)$ for some integer $N > 1$. So, there exists $x \in l_\infty$, such that $\mathcal{B}x \notin l_\infty$. Now, $y = (y_k) = (N^{\frac{1}{pk}} k^{\frac{s}{pk}} x_k) \in l_\infty(p.s)$, but $\mathcal{A}y = \mathcal{B}x \notin l_\infty$, which contradicts the fact that $\mathcal{A} \in (l_\infty(p.s), l_\infty)$. Hence 2.1 is necessary.

Sufficiency. Let the condition (2.1) holds and $x \in l_\infty(p.s)$. If we choose an integer $N > max(1, \sup_k k^{-s}|x_k|^{p_k})$, then for every n,i.

$$\left|(Ax)_n^i\right| = \left|\sum_k a_{nk}^{(i)} x_k\right| = \sum_k \left|a_{nk}^{(i)} x_k\right| = \sum_k \left|a_{nk}^{(i)}\right| k^{\frac{s}{p_k}} N^{\frac{1}{p_k}} \leq D(N).$$

And therefore $\mathcal{A} \in (l_\infty(p.s), l_\infty)$. This completes the proof of the Theorem 2.1. □

Theorem 2.3. *(a) $\mathcal{A} \in (l_\infty(p.s).f)$ if and only if (2.1) holds and,*

$$f - lim a_{nk}^{(i)} = a_k, \ \textit{uniformly in i, for each k}, \tag{2.2}$$

$$\lim_q \sum_k \frac{1}{q} + 1 \left|\sum_{m=0} a_{n+m,k}(i) - a_k\right| = 0, uniformlyinn, i. \tag{2.3}$$

(b) $\mathcal{A} \in (l_\infty(p,s), f_0)$ if and only if (2.1) holds and (2.2), (2.3) also hold with $a_k = 0$ for each k, where f_0 denotes the linear space of all almost convergent sequences whose generalized limit is zero.

To prove the Theorem 2.3 we require the following result (see Basar Solak[3]).

Lemma 2.4. *$\mathcal{A} \in (l_\infty, f)$ if and only if (2.2), (2.3) hold and $\sup_{n,i} \sum_k \left|a_{nk}^{(i)}\right| < \infty$.*

Proof of the Theorem 2.3 (a). Necessity. Let $\mathcal{A} \in (l_\infty(p.s), f)$. Since $(l_\infty(p,s), f) \subset (l_\infty(p.s), l_\infty)$, the condition (2.1) must hold.the necessity of (2.2) is obtained by taking $x = e^k$, where e^k is the sequence whose only non-zero term is 1 in the k-th place. Since $e^k \in l_\infty(p.s)$ for each k. If (2.3) is not true,then by Lemma 2.4, the matrix sequence $\mathcal{B} = (a_{nk}^{(i)} k^{\frac{s}{pk}} N^{\frac{1}{pk}}) \notin (l_\infty, f)$ for some integer $N > 1$. So that there exists $x \in l_\infty$ such that $\mathcal{B}x \notin f$. Let us take $y = (k^{\frac{s}{pk}} N^{\frac{1}{pk}} x_k) \in l_\infty(p.s)$, but $\mathcal{A}y = \mathcal{B}x \notin f$. This contradicts the fact that $\mathcal{A} \in (l_\infty(p.s), f)$. Hence (2.3) is true.

Sufficiency: Suppose that the conditions (2.1)-(2.3) hold and $x \in l_\infty(p.s)$. Since $x \in l_\infty(p.s)$ and the series $\sum a_k$ converges absolutely, the series $\sum a_k x_k$ also converges absolutely,say to the value b_0. By using similar technique as in [3], we get by (2.3).

$$0 \le \lim_q \left| \frac{1}{q} + 1 \sum_{m=0}^{q} (Ax)^i_{n+m} b_0 \right| = \lim_q \frac{1}{q+1}$$

$$\left| \sum_{m=0}^{q} \sum_k (a_{n+m,k}(i) - a_k) x_k \right| \le \lim_q \sum_k \frac{1}{q+1}$$

$$\left| \sum_{m=0}^{q} a_{n+m,k}(i) - a_k \right| |x_k| \le K lim \sum_k \frac{1}{q+1}$$

$$\left| \sum_{m=0}^{q} a_{n+m,k}(i) - a_k \right| = 0,$$

uniformly in n,i. This means that $f - lim \mathcal{A}x = b_0$, uniformly in n,i and hence $\mathcal{A} \in (l_\infty(p.s), f)$. Proof of (b) follows from (a) by taking $a_k = 0$ for each k. This completes the proof of the Theorem 2.3. □

Theorem 2.5. *(a) $\mathcal{A} \in (l_\infty(p.s).bs)$ if and only if (2.4) holds. For every integer $N > 1$,*

$$\sup_{n,i} \sum_k \left| \sum_{j=0}^{n} a_{jk}(i) \right| k^{\frac{s}{p_k}} N^{\frac{1}{p_k}} < \infty. \tag{2.4}$$

Proof. Let $x \in l_\infty(p.s)$. Consider the following equality obtained from the m-th partial sums of $\sum_{j=0}^{m} (Ax)^i_j$,

$$\sum_{j=0}^{n} \sum_{k=0}^{m} a_{jk}(i) x_k = \sum_{k=0}^{m} \left(\sum_{j=0}^{n} a_{jk}(i) \right) x_k : m, n, i = 0, 1... \tag{2.5}$$

we get by leting $m \to \infty$ in (2.5) that

$$\sum_{j=0}^{n} \sum_k a_{jk}(i) x_k = \sum_k \left(\sum_{j=0}^{n} a_{jk}(i) \right) x_k : n, i = 0, 1... \tag{2.6}$$

Thus it is seen in (2.6) that $\mathcal{B} = b_{nk}(i) \in (l_\infty(p.s).l_\infty)$, where

$$b_{nk}(i) = \sum_{j=0}^{n} a_{jk}(i)$$

for all n,k and i. Therefore $\mathcal{B} \in (l_\infty(p.s).l_\infty)$, if and only if $\mathcal{A} \in (l_\infty(p.s).bs)$. This completes the proof of Theorem 2.5. □

Theorem 2.6. *(a) $\mathcal{A} \in (l_\infty(p.s).fs)$ if and only if (2.4) holds and*

$$f - lim \sum_{j=0}^{n} a_{jk}(i) = a_k, \textit{uniformly in } i \textit{ for each } k, \tag{2.7}$$

$$\lim_q \sum_k \frac{1}{q+1} | \sum_{m=0}^{q} \sum_{j=0}^{n+m} a_{jk}(i) - a_k| = 0, \textit{uniformly in } n,i. \tag{2.8}$$

(b) $\mathcal{A} \in (l_\infty(p.s), f_0 s)$ if and only if (2.4) holds and (2.7), (2.8) also hold with $a_k = 0$ for each k, where $f_0 s$ denotes the linear space of all almost convergent series whose generalized sum is zero.

Proof (a). Necessity. Let $\mathcal{A} \in (l_\infty(p.s).fs)$ and $x \in l_\infty(p.s)$ the necessity of (2.7) is proved by taking $x = e^k$. Now, reconsider the equality (2.6). It is seen by passing f-limit in (2.6) that $\mathcal{B} = (b_{nk}(i)) \in (l_\infty(p.s).f)$, where

$$b_{nk}(i) = \sum_{j=0}^{n} a_{jk}(i) \quad for \ \ all \ \ n, k \ \ and \ \ i.$$

Therefore, $\mathcal{B} = b_{nk}(i)$ satisfies (2.1), (2.3) and these are equivalent to (2.4), (2.8) respectively.

Sufficiency: Suppose (2.4), (2.7), (2.8) hold and $x \in l_\infty(p.s)$. Again consider

$$\mathcal{B} = \left(\sum_{j=0}^{n} a_{jk}(i) \right)$$

In (2.6). Therefore, it follows immediately that $\mathcal{B} = b_{nk}(i)$ satisfies (2.1), (2.2) and (2.3) if and only if $\mathcal{A} = a_{nk}(i)$ satisfies (2.4), (2.7) and (2.8) respectively. Hence $\mathcal{B} \in (l_\infty(p.s).f)$, and this yields, by passing f-limit in (2.6) that $x \in fs$. Then $\mathcal{A} \in (l_\infty(p.s).fs)$,.

Proof of (b) follows from (a) by taking $a_k = 0$ for each k. This completes the proof of Theorem 2.6. □

3 Matrix sequences from bs(p,s) into $l_\infty.f.bs$ and fs.

Theorem 3.1. *$\mathcal{A} \in bs(p,s).l_\infty$ if and only if (3.1), (3.2) holds. For every integer $N > 1$*

$$\sup_{n,i} \sum_k |\Delta a_{nk}(i)| k^{\frac{s}{p_k}} N^{\frac{1}{p_k}} < \infty. \tag{3.1}$$

$$\lim_k a_{nk}(i) = 0 \textit{for each } n,i. \tag{3.2}$$

Where $\Delta a_{nk}(i) = a_{nk}(i) - a_{n,k+1}(i)$.

Proof. Necessity. Let $\mathcal{A} \in bs(p,s).l_\infty$ and $x \in bs(p.s)$. To show the necessity of (3.2), we assume that (3.2) is not true for some n,i and obtain a contradiction as in Theorem 2.1 of [11]. Indeed under this assumption we can find some $x \in bs(p,s)$ such that $\mathcal{A}x \in l_\infty$. For example of we choose $x = (-1)^n \in bs(p,s)$ then

$$(\mathcal{A}x)_n^i = \sum_k a_{nk}(i)(-1)^k$$

However,the series

$$\sum_k a_{nk}(i)(-1)^k$$

does not converge for each n,i. That is to say that $\mathcal{A}$-transform of the series $\sum(-1)^n$ which belongs to bs(p,s) does not even exist. But this contradicts the fact that $\mathcal{A} \in bs(p,s).l_\infty$. Hence (3.2) is necessary.

Let us consider the equality

$$\sum_{k=0}^{m} a_{nk}(i)x_k = \sum_{k=0}^{m-1} \Delta a_{nk}(i)s_k + a_{nm}(i)s_m, \quad : m,n,i = 0,1... \tag{3.3}$$

obtained by applying Abel's partial summation to the m-th partial sums of $\mathcal{A}$x.By letting $m \to \infty$ in (3.3) and from (3.2),we have

$$\sum_k a_{nk}(i) = \sum_k \Delta a_{nk}(i)s_k \quad : n,i = 0,1... \tag{3.4}$$

Thus,it is that seen that $\mathcal{C} = c_{nk}(i) \in (l_\infty(p,s), l_\infty)$ where $c_{nk}(i) = \Delta a_{nk}(i)$, for all n,k and i. Therefore $\mathcal{C} = c_{nk}(i)$ satisfies (2.1) which is equivalent to (3.1).

Sufficiency: Suppose that the conditions (3.1), (3.2) hold and $x \in bs(p,s)$. Now reconsider $\mathcal{C} = c_{nk}(i)$ in (3.4). Therefore $\mathcal{C} = c_{nk}(i)$ satisfies (2.1) if and only if $\mathcal{A} = a_{nk}(i)$ satisfies (3.1) is true. So $\mathcal{C} \in (l_\infty(p,s), l_\infty)$. This implies by (3.4) that $\mathcal{A} \in bs(p,s)).l_\infty)$. This completes the proof of Theorem 3.1. □

Theorem 3.2. *(a) $\mathcal{A} \in bs(p.s).f$ if and only if (3.1), (3.2), (3.5) and (3.6) hold.*

$$f - lim a_{nk}(i) = a_k, \textit{uniformly in } i \textit{ for each } k. \tag{3.5}$$

$$\lim_q \sum_k \frac{1}{q} + 1 | \sum_{j=0}^{q} (\Delta a_{n+j,k}(i) - a_k)| = 0, \textit{uniformly in n,i.} \tag{3.6}$$

(b) $\mathcal{A} \in bs(p.s).f_0$ if and only if (3.1), (3.2) hold and (3.5), (3.6) also hold with $a_k = 0$ for each k.

Proof (a). Necessity. Suppose $\mathcal{A} \in (bs(p,s), f)$. Since (bs(p,s),f)$\subset (bs(p,s), l_\infty)$, the necessity of (3.1) and (3.2) are obvious. The necessity of (3.6) is easily proved in the same way as was (2.2), with bs(p,s) instead of $l_\infty(p,s)$.

Now, reconsider the equality (3.4). It is seen by passing to f-limit in (3.4) that $\mathcal{B} = (b_{nk}(i)) \in (l_\infty(p.s).f)$, where $b_{nk}(i) = \Delta a_{nk}(i)$ for all n,k and i. Therefore $\mathcal{B} = (b_{nk}(i))$, satisfies (2.3) which is equivalent to (3.6).

Sufficiency: Suppose that the conditions (3.1), (3.2), (3.5), (3.6) hold and $x \in bs(p,s)$. Let us reconsider $\mathcal{B} = (\Delta a_{nk}(i))$ in (3.4). Therefore $\mathcal{B} = a_{nk}(i)$ satisfies (2.1), (2.2) and (2.3) if and only if $\mathcal{A} = a_{nk}(i)$ satisfies (3.1), (3.2), (3.5) and (3.6) respectively is true. Hence $\mathcal{B} \in (l_\infty(p,s), f)$ and this yields by passing f-limit in (3.4) that $\mathcal{A}x \in f$. This asserts that every element of bs(p,s) is almost A-summable i.e, $\mathcal{A} \in (l_\infty(p,s), f)$.

Proof of (b) follows from (a) by taking $a_k = 0$ for each k. This completes the proof of Theorem 3.2.

Theorem 3.3. *$\mathcal{A} \in (bs(p,s), bs)$ if and only if (3.2)and(3.7) hold. For every integer $N > 1$,*

$$\sup_{n,i} \sum_k \left| \sum_{j=0}^{n} \Delta a_{jk}(i) \right| k^{\frac{s}{pk}} N^{\frac{1}{pk}} < \infty. \tag{3.7}$$

Proof (a). Necessity. Let $\mathcal{A} \in (bs(p,s), bs)$ and $x \in bs(p,s)$. Since $(bs(p,s), bs) \subset (bs(p,s), l_\infty)$, the necessity of (3.2) is obvious Theorem 3.1.

Now, consider the equality, which is obtained in a similar way of (3.3);

$$\sum_{j=0}^{n} \sum_{k=0}^{m} a_{jk}(i) x_k = \sum_{k=0}^{m-1} (\sum_{j=0}^{n} \Delta a_{jk}(i)) s_k + \sum_{j=0}^{n} a_{jm}(i) \quad s_m : m, n, i = 0, 1... \tag{3.8}$$

Therefore, we get by considering (3.2) and letting $m \to \infty$ in (3.8) that

$$\sum_{j=0}^{n} \sum_k a_{jk}(i) x_k = \sum_k (\sum_{j=0}^{n} \Delta a_{jk}(i)) s_k : n, i = 0, 1... \tag{3.9}$$

Thus it is seen that $\mathcal{B} = b_{nk}(i) \in (l_\infty(p.s).l_\infty)$, where $b_{nk}(i) = \sum_{j=0}^{n} \Delta a_{jk}(i)$ for all n,k and i. So $\mathcal{B} = b_{nk}(i)$ satisfies (2.1)which is equivalent to (3.7).

The sufficiency is trivial. This completes the proof of Theorem 3.3.

Theorem 3.4. *(a) $\mathcal{A} \in (bs(p.s).fs)$ if and only if (3.2), (3.7), (3.10) and (3.11) hold.*

$$f - lim \sum_{j=0}^{n} a_{jk}(i) = a_k, \textit{uniformly in } i \textit{ for each } k. \tag{3.10}$$

$$\lim_q \sum_k \frac{1}{q+1} \left| \sum_{j=0}^{q} \sum_{m=0}^{n+j} \Delta(a_{nk}(i) - (a_k)) \right| = 0, \textit{uniformly in } n,i. \tag{3.11}$$

(b) $\mathcal{A} \in (bs(p.s), f_0 s)$. if and only if (3.2), (3.7) hold and (3.10), 3.11) also hold with $a_k = 0$ for each k.

Proof (a). Necessity. Let $\mathcal{A} \in (bs(p,s), fs)$ and $x \in bs(p,s)$. Since (bs(p,s),fs)⊂ $(bs(p,s), bs)$, the necessity of (3.2) and (3.7) are obvious by Theorem 3.3. The necessity of (3.10) is proved by the analogous argument of (3.5).

Now, reconsider the equality (3.9). It is seen by passing f-limit in (3.9) that $\mathcal{B} = b_{nk}(i) \in (l_\infty(p.s).f)$, where

$$b_{nk}(i) = \sum_{j=0}^{n} \Delta a_{jk}(i) \quad for \;\; all \;\; n, \; k \;\; and \;\; i.$$

So $\mathcal{B} = b_{nk}(i)$ satisfies (2.3) which is equivalent to (3.11).

The sufficiency and the proof of (b) is obvious. This completes the proof of Theorem 3.4.

Acknowledgement(s) : The authors are thankful to Prof.Mursaleen for his valuable suggestions throughout the preperation of this paper. The authors thank the Referee for his suggestions and comments.

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(Received 28 August 2009)

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