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DIGITAL IMAGE RESTORATION : A COMPARISON STUDY BETWEEN INVERSE AND WEINER FILTERING ALGORITHMS (IWFA)

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Abstract In this article, pictures model are used in education. Digital image restoration by graying the image using effects, Motion Blurred Image, animation and image deterioration patterns that can be controlled according to the conditions we created with. Random disturbances through the Scilab and Mathlab program. Images that are blurred or deteriorated by using Inverse and Weiner Filtering to compare the return of the image to return to the original image using the time and efficiency of the original. It is a measure.

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Keywords: Image Restoration; Blur model; Noise model; Inverse Filtering; Weiner Filtering

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1. Introduction

In 1950, space scientists captured images from the surface of the Earth, which were very challenging to shoot on space. That is when shooting on space Shooting may result in image perfection, misalignment or image distortion. However, the movement of the lens may cause blur or blurry images. Which the goal of restoring the image is to be close to the original image or to improve the problematic image by using a mathematical program to restore the problematic image, which the image restoration uses to be approximated to Model created In the same way, bringing degraded images to improve or bring good images to improve better than ever is a very challenging thing (see [8, 12–20]).

However, image restoration is widely used in science, astronomy, images recorded with closed-circuit television. Or medical imaging such as x-rays images etc.

In this research, the model will be created to restore good images by using inverse

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and wiener filtering by converting the image to grayscale with visual disturbances. Then restore the blurred image using weiner filtering as a filter to refresh the image.

2. Image Restoration

The image restoration is to make the image as close to the original image as possible. However, image deterioration occurs during image reception or image transmission. Mainly depends on the environment Light moisture And temperature etc. The main objective of image restoration is to improve the quality of digital images to be effective as in Figure 1.

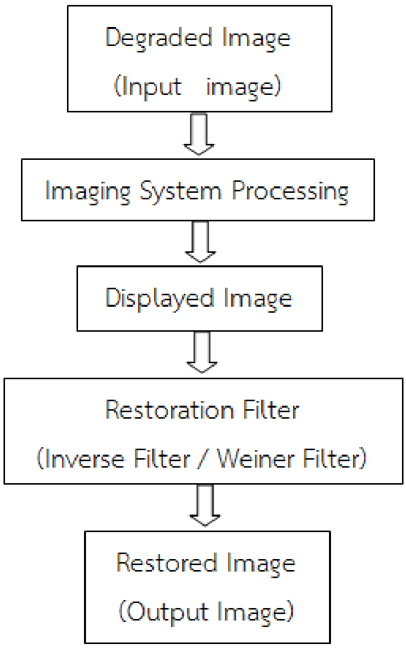


Figure 1. The Flow Chart of the restoration model

The original input is a two-dimensional image $f(x, y)$. This image is operated on by the system H and after the addition of $\xi(x, y)$ one can obtain the degrade image $g(x, y)$. Digital image restoration may be viewed as a process in which we try to obtain an approximation to $f(x, y)$ given $g(x, y)$ and H and after applying Restoration filters we obtain restored image $\hat{f}(x, y)$. The degradation phenomenon is mathematically expressed as Figure 2.

In the spatial domain, the degradation of the original image can be modeled as

$$g(x, y) = h(x, y) \oplus f(x, y) + \xi(x, y), \tag{2.1}$$

where (x, y) is the detached pixel coordinates of the image frame.
$f(x, y)$ is the original image
$g(x, y)$ is the degraded image
$h(x, y)$ is the image degradation function
$\xi(x, y)$ is the ad-on noise.

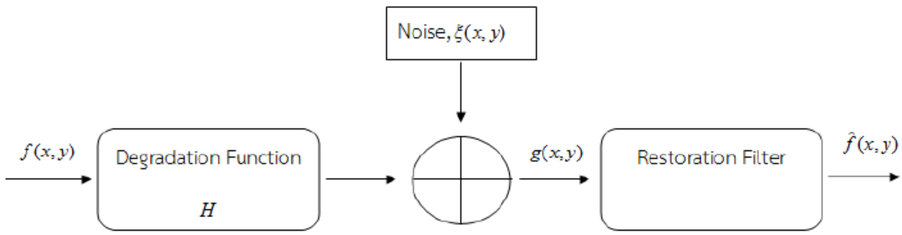


Figure 2. The Flow Chart of the image degradation and restoration procedure

2.1. Noise model

The noise of the noise is an unpleasant form that occurs from the photograph, resulting in a change in the vision of the image. Which the main cause of image interference occurs during image reception or image sensor Causing damage to images in the form of Gaussian sounds, salt, and pepper sounds (Impulse sound) as following.

Gaussian noise

This noise is typical in sensors, especially in low lighting condition or poor illumination. Gaussian noise is as Figure 3 and Figure4.

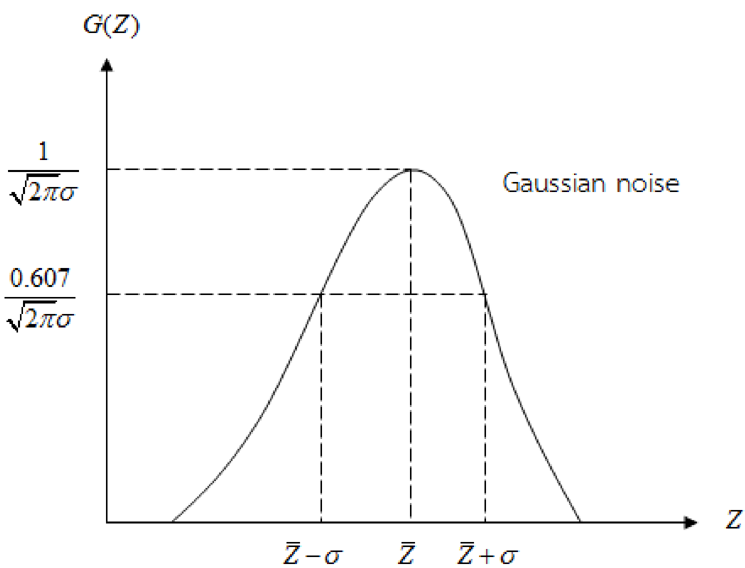


Figure 3. The Gaussian noise 2 dimension

FIGURE 4. The Gaussian noise digital image

Gaussian noise is characterized by two parameters, μ (mean) and σ^2 (variance). Remote sensing images are affected by multiplicative noise in addition to additive noise. In remote sensing system sensors, the scattered waves constructively or destructively interfere with each other causing a speckled appearance to the image. This type of noise can be given as a unity-mean the normal random process given by

$$G(Z) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(Z-\mu)^2}{2\sigma^2}}, \tag{2.2}$$

where g is the gray level
μ is the mean
σ is the standard deviation.

Salt and pepper noise (Impulse noise)

Salt-and-pepper noise is a form of noise sometimes seen on images. It is also known as impulse noise. This noise can be caused by sharp and sudden disturbances in the image signal presents itself as sparsely occurring white and black pixels

$$\text{Salt and Pepper} = \begin{cases} A \; for \; g = a & \text{Pepper noise} \\ B \; for \; g = b & \text{Salt noise} \\ 0 & \text{Otherwise.} \end{cases}$$

If either A or B is zero, the impulse noise is called unipolar. a and b usually are extreme values because impulse corruption is usually large compared with the strength of the image signal. It is the only type of noise that can be distinguished from others visually as Figure 5.

FIGURE 5. The Gaussian noise Salt and pepper noise

Uniform noise

The level of the gray values of the noise is uniformly distributed across a specified range in uniform noise. Speckle noise mostly occurred in remote sensing images at the time of image acquisition. The mean and variance of this density is given

$$P(Z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq Z \leq b \\ 0 & \text{Otherwise,} \end{cases}$$

where $\mu = \frac{a+b}{2}$ and $\sigma^2 = \frac{(b-a)^2}{12}$.

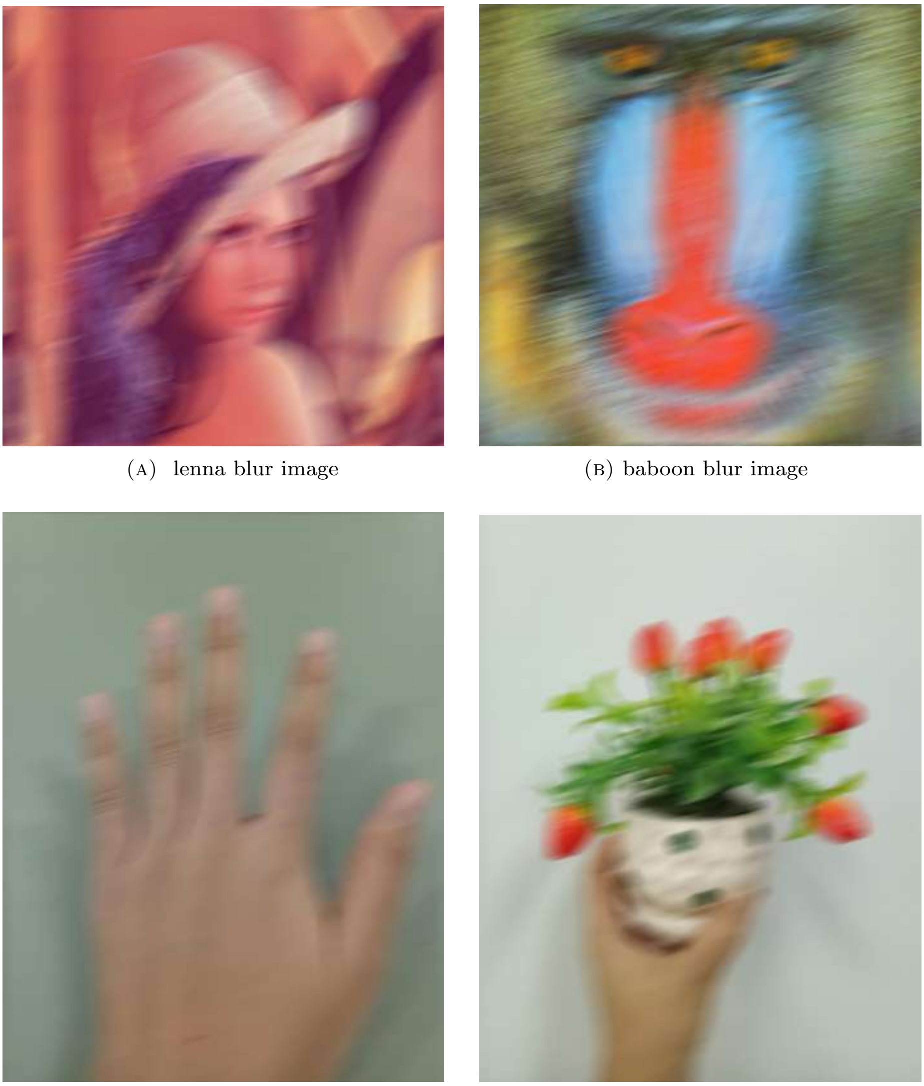
2.2. Blur model

Gaussian blur

It is type of image blurring filter which use Gaussian function for calculating transformation applied on each pixel. The equation of Gaussian function is

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}, \tag{2.3}$$

where (x, y) is distance from origin in horizontal axis and σ is standard deviation of Gaussian distribution as Figure 6.



(A) lenna blur image

(B) baboon blur image

(C) hand blur image

(D) flower blue image

FIGURE 6. The image degradation and restoration using Gaussian blur

Motion blur

The motion of the image can occur when the camera fires or is caused by the movement of the subject, which may cause blurred images as Figure 7.

Figure 7. The motion blur image

2.3. Point Spread Function

It is also known as "Point Spread Functions" or PSF is a diffusion function based on the "Airy Pattern" diffraction theory, which uses all-optical models to calculate point out-of-focus distribution functions when the function of the point spread through the focus has an abnormality.

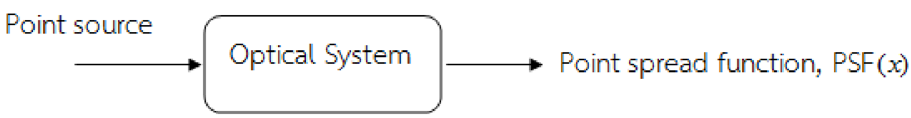


Figure 8. The Flow Chart of the Point Spread Function

The point spread function of human eyes as following.

2.4. Inverse Filtering

Inverse filtering is a linear filter. If we know the exact point-spread function model of the image degradation system and noise effect is negligible, then degraded image can be restored using the inverse filter approach.

The main advantage of the inverse filter is that it is straightforward and simple. The second advantage, it is perfectly reconstructed in the absence of noise and it requires only the blur point spread function (PSF) as a priori knowledge. The disadvantage of the inverse filter is that it will not perform well in the presence of noise. If there is a noise in the image, the inverse filter tends to amplify the undesirable noise. In the presence

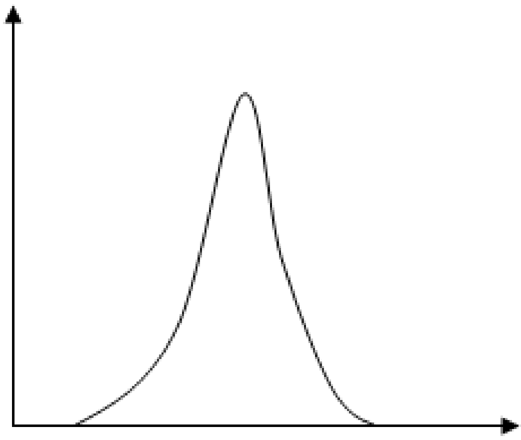


FIGURE 9. The Flow Chart of the point spread function of human eyes

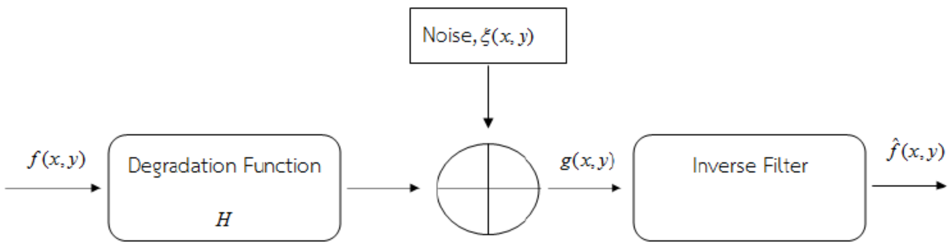


FIGURE 10. The Flow Chart of the Inverse Filtering

of noise, it is better to go for a Weiner filter. The second disadvantage, it is not always possible to obtain an inverse filter is given as

$$\hat{F}(u,v) = \frac{1}{H(u,v)} \tag{2.4}$$

and the recovered image can be expressed as

$$\hat{F}(u,v) = G(u,v)\hat{H}(u,v) \tag{2.5}$$

Pseudo Inverse Filter

Pseudo inverse filter can be expressed as

$$\hat{H}(u,v) = \begin{cases} \frac{1}{H(u,v)} & if \quad |H(u,v)| \geq k \\ 0 & if \quad |H(u,v)| < k. \end{cases}$$

(A) Degraded Image

(B) Restored Image

(C) Degraded Image

(D) Restored Image

FIGURE 11. The Image degradation and restoration using inverse filter

(A) Degraded Image

(B) Inverse Filter

FIGURE 12. The image degradation and restoration using pseudo inverse filter

(A) Blurred Image

(B) Noisy with degraded Image

(C) Psedo-inverse restoration

FIGURE 13. The image degradation and restoration using pseudo inverse filter

2.5. Weiner Filtering

In the 1950s it was known as the Weiner filtering algorithm. It was an algorithm used to process unwanted images. Therefore, this algorithm is very popular in the restoration of images.

Theorem 2.1. *Parseval's theorem*
Its aim is to produce an estimate of the underlying image such that the expected mean square error between the true and estimated images is minimized:

$$\begin{aligned} e^2(\hat{f}(x,y)) &= E(\int\int (f(x,y) - \hat{f}(x,y))^2 \, dxdy) \\ &= E(\int\int (F(u,v) - \hat{F}(u,v))^2 \, dudv). \end{aligned}$$

Theorem 2.2. *Estimate of the Fourier transform of f*

$$\hat{F}(u,v) = [\frac{1}{H(u,v)} \ \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}]G(u,v),$$

where

$H(u,v)$ is the degradation function

$|H(u,v)|^2 = H^*(u,v)H(u,v)$ with $H^*(u,v)$ the complex conjugate of H

$S_\eta(u,v) = |N(u,v)|^2$ is the power spectrum of the noise

$S_f(u,v) = |F(u,v)|^2$ is the power spectrum of the undergraded image.

If the noise spectrum $|N(u,v)|^2$ is zero, the noise-to-signal power ratio $\frac{S_\eta(u,v)}{S_f(u,v)} = \frac{|N(u,v)|^2}{|F(u,v)|^2} = 0$ vanishes and the Weiner Filter reduces to the inverse filter.

$$\begin{aligned}\hat{F}(u,v) &= [\frac{1}{H(u,v)}\ \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}]G(u,v)\\ &= [\frac{1}{H(u,v)}\ \frac{|H(u,v)|^2}{|H(u,v)|^2 + 0}]G(u,v)\\ \hat{F}(u,v) &= \frac{1}{H(u,v)}G(u,v).\end{aligned}$$

This is no problem, as the inverse filter works fine if no noise is present. The main problem with the Weiner filter is that the power spectrum $|F(u,v)|^2$ of the undegraded image is seldom known. A frequently used approach when these quantities cannot be estimated is to approximate by the expression

$$\hat{F}(u,v) = [\tfrac{1}{H(u,v)}\ \tfrac{|H(u,v)|^2}{|H(u,v)|^2+K}]G(u,v),$$

where

$\hat{F}(u,v)$ is Estimate of the Fourier transform of f

$S_\eta(u,v)$ is power spectum of original signal $|F(u,v)|^2$

$S_f(u,v)$ is power spectum of noise $|N(u,v)|^2$

$\frac{S_\eta(u,v)}{S_f(u,v)}$ is noise to signal ratio (or inverse signal to noise ratio: ISNR)

$\frac{1}{H(u,v)}$ is deblurring filter.

The Weiner filter is to locate an approximation $\hat{f}(u,v)$ of original image $f(u,v)$ so as to the mean square error between them is minimized. weiner filter is represented as $W(u,v)$ as shown below.

$$W(u,v) = \tfrac{H^*(u,v)}{|H(u,v)|^2 + \frac{P_n(u,v)}{P_f(u,v)}}.$$

Againt, we assume

$$W(u,v) = \frac{1}{H(u,v)}\ \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}, \tag{2.6}$$

where

$K = \frac{P_n(u,v)}{P_f(u,v)}$

$P_n(u,v)$ is Noise power spectum

$P_f(u,v)$ is power spectum of the original image

Here, K is the inverse of the Peak Signal to Noise Ratio (PSNR).

The Weiner filter can generate optimal estimate only if such stochastic processes are stationary Gaussian. This situations are not typically satisfied for real image. So the restored image can be expressed as :

$$\hat{f}(u, v) = W(u, v)G(u, v)$$

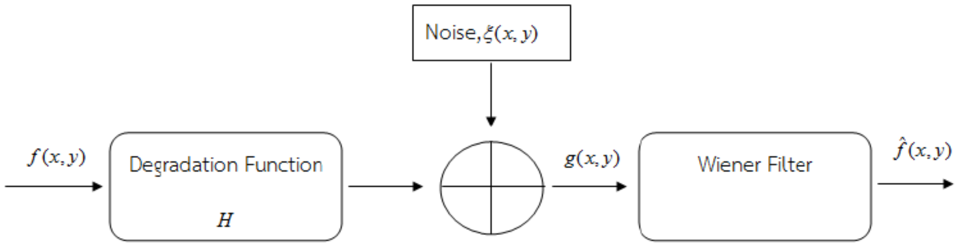
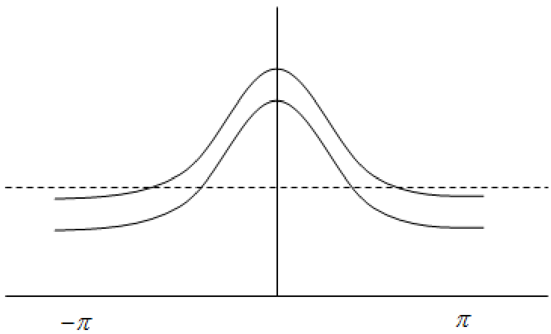
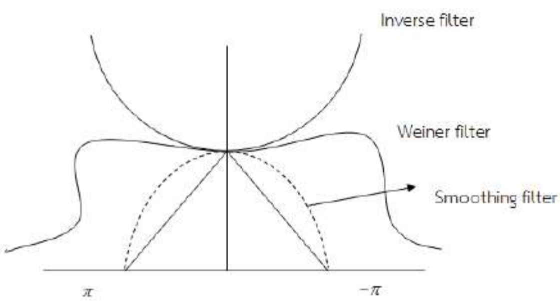


FIGURE 14. The Flow Chart of the Weiner Filtering



(A) Noise smoothing



(B) Deblurring

FIGURE 15. The Flow Chart of image degradation and restoration using Weiner filtering

2.6. Inverse and Weiner Filtering Algorithm (IWFA)

In this research, we apply the concepts of unwanted images such as blurred images, old images, etc. by using these images to make them impressive. We use the techniques of the inverse filter and Weiner filter to support this research, which shows the diagram. Follow this site.

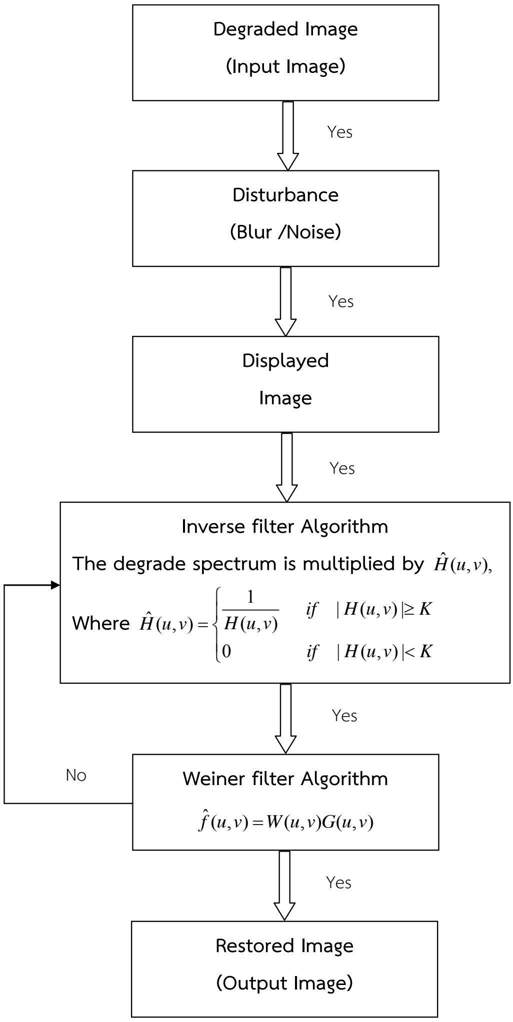


Figure 16. The Flow Chart of the inverse and wiener filtering algorithm

2.7. Performance Parameters

For performance parameters, various medical image were considered of different type and size 256 × 256. Performance Metrices are MSE (Mean Square Error) and PSNR (Peak Signal to Noise Ratio).

MSE: It is defined as the difference between the actual and the estimated signals. It is expressed as follows.

$$MSE = E[\{f(x,y) - \hat{f}(x,y)\}^2]$$

or

$$MSE = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}[x(i,j) - y(i,j)]^2, \tag{2.7}$$

where $x(i,j)$ represents the original (reference) image and $y(i,j)$ represents the distorted (modified) image and i and j are MSE is zero when $x(i,j) = y(i,j)$.

PSNR : It is defined as the Peak Signal to Noise Ratio (PSNR): The PSNR is evaluated in decibels and is inversely proportional the Mean Squared Error. PSNR is expressed in terms of decibel (dB).It is expressed mathematically as follows:

$$PSNR = 10\log_{10}\frac{(2^n - 1)^2}{\sqrt{MSE}}, \tag{2.8}$$

where $2^n - 1$ is maximum fluctuation in input image data type and PSNR measures the peak error.

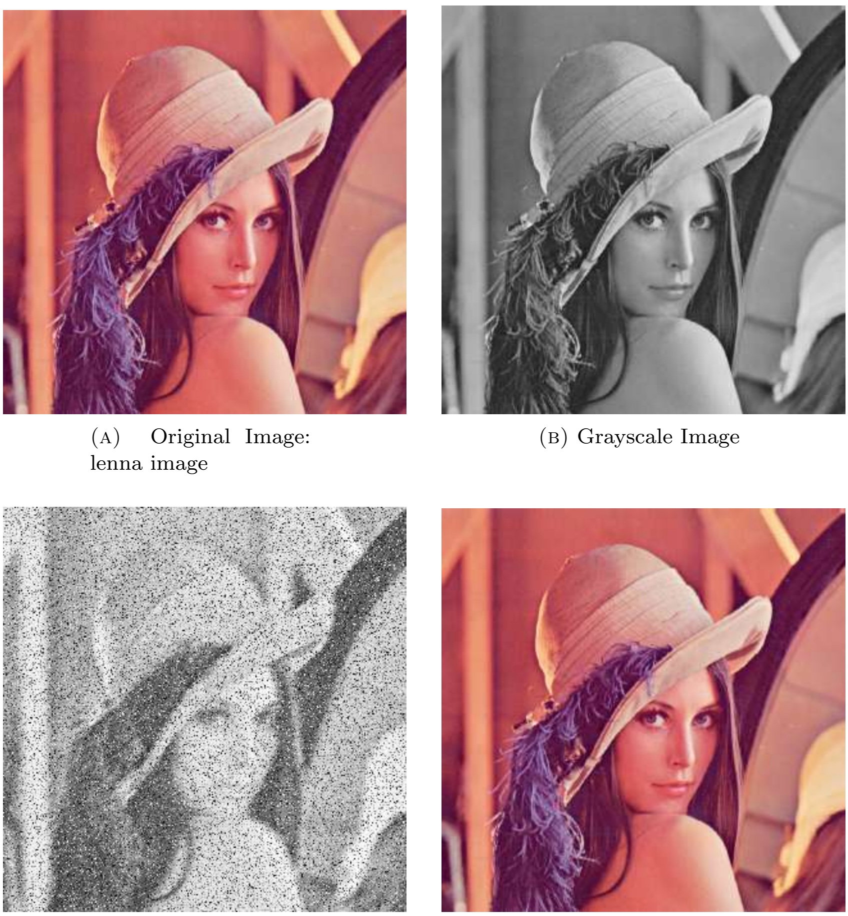
SNR : computed as the Signal to Noise Ratio of the corresponding pixels in reference and restored images. Its value will be higher when reference image and restored image are alike. Higher value implies better restoration (V.P.S Naidu, 2008).

$$SNR = 10\log_{10}\left(\frac{\sum_{i=1}^{M}\sum_{j=1}^{N} I_r(i,j)^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}(I_r(i,j) - I_f(i,j))^2}\right). \tag{2.9}$$

3. Results and Discussion

3.1. The results of Inverse Filtering

For inverse filtering, if we do not add any noise after the motion blur, then we can restore the same image before motion blur. The figure below shows the effectiveness of inverse filtering without any noise. Now, if we add some random noise to the image, then the filter performance degrades to some extent. The consequence of noise on the performance of inverse filtering is made known in Figure 17, 18, 19 and 20. In Figure 17, 18, 19 and 20, though the inverse filter is capable of inverting the effect of motion blur, it is not able to nullify the effect of noise. Here, $a = 0.001$ and $b = 0.01$.



(A) Original Image: lenna image

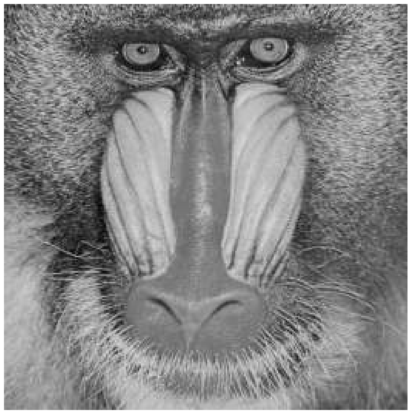
(B) Grayscale Image

(C) Salt and Pepper/Gaussian/ Motion Blurred Image

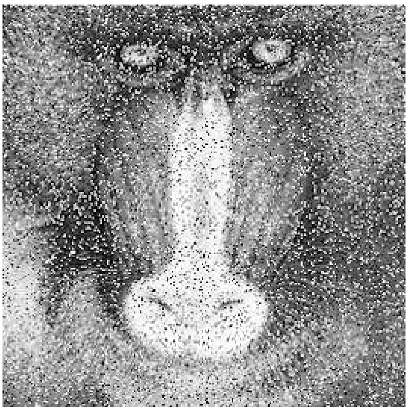
(D) Restored Image

Figure 17. The image degradation and restoration using Inverse filter

(A) Original Image: baboon image



(B) Grayscale Image



(C) Salt and Pepper/Gaussian/ Motion Blurred Image

(D) Restored Image

FIGURE 18. The image degradation and restoration using Inverse filter

(A) Original Image: hand image

(B) Grayscale Image

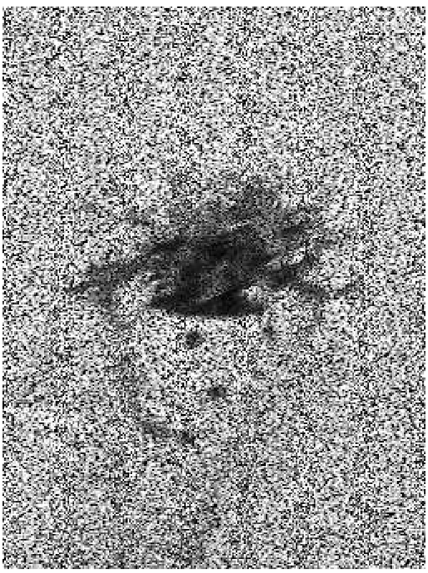
(C) Salt and Pepper/Gaussian/ Motion Blurred Image

(D) Restored Image

FIGURE 19. The image degradation and restoration using Inverse filter

(A) Original Image: hand image

(B) Grayscale Image



(C) Salt and Pepper/Gaussian/ Motion Blurred Image

(D) Restored Image

FIGURE 20. The image degradation and restoration using Inverse filter

3.2. The results of Weiner Filtering

The winer filter has a "K" component which is inverse to the PSNR. Now, if the noise power is zero, which means no noise, then the weiner can restore the exact image which was corrupted by salt and pepper/Gaussian/ motion blurred image effect.And we change the "K" to 0.001. In the following case, we have considered zero noise power, and Figure 21, 22, 23 and 24 shows the performance of the Weiner filter.

(A) Original Image: lenna image

(B) Restored process

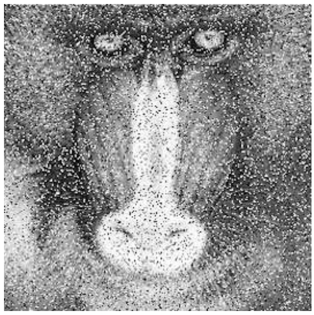
(C) Restored process

(D) Restored process

(E) Restored Image

FIGURE 21. The image degradation and restoration using Weiner filter

(A) Original Image: baboon image



(B) Restored process

(C) Restored process

(D) Restored process

(E) Restored Image

FIGURE 22. The image degradation and restoration using Weiner filter

(A) Original Image: hand image

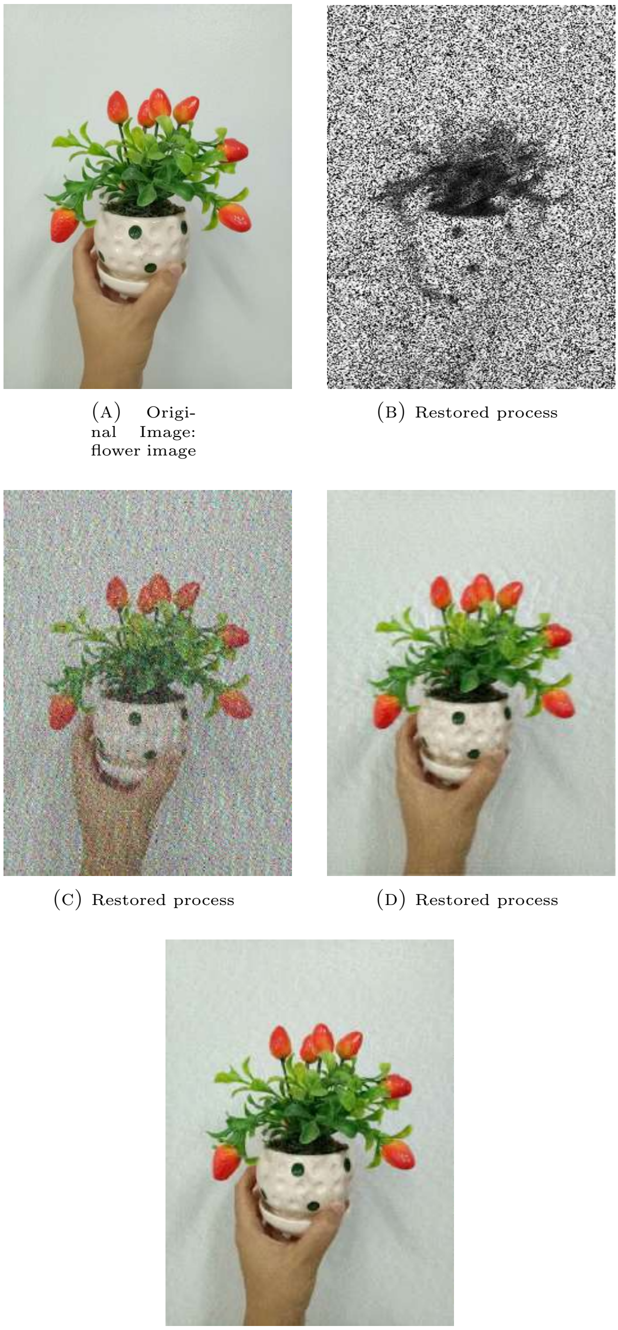
(B) Restored process

(C) Restored process

(D) Restored process

(E) Restored Image

FIGURE 23. The image degradation and restoration using Weiner filter



(A) Original Image: flower image

(B) Restored process

(C) Restored process

(D) Restored process

(E) Restored Image

FIGURE 24. The image degradation and restoration using Weiner filter

4. Experimental results

Performance Measures							
		Inverse Filter			Weiner Filter		
No.	Images	MSE	PSNR	Time(s)	MSE	PSNR	Time(s)
1	lenna	0.05299	45.9624	1.2568	0.04731	45.8216	1.6789
2	baboon	0.03975	44.8628	1.1546	0.02847	44.9256	1.4821
3	hand	0.09167	47.7781	1.3187	0.08631	47.6235	1.8931
4	flower	0.07569	49.2561	1.5227	0.06741	49.1482	1.9624

5. Conclusions

In this paper, we have studied Digital Image Restoration: A Comparison Study between Inverse and Weiner Filtering Algorithms (IWFA) the model of a picture are used in education. Digital image restoration by graying the image using effects, Motion Blurred Image, animation and image deterioration patterns that can be controlled according to the conditions we created with. Random disturbances through the Scilab and Mathlab program. Images that are blurred or deteriorated by using Inverse and Weiner Filtering to compare the return of the image to return to the original image and the results of the test, 4 test images, Lenna, baboon, hand and flower image, found that the performance measures of the inverse filter Weiner filter were similarly effective with using the time and mean square error It is a measure.

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